

Spatial Lags and Spatial Correlograms

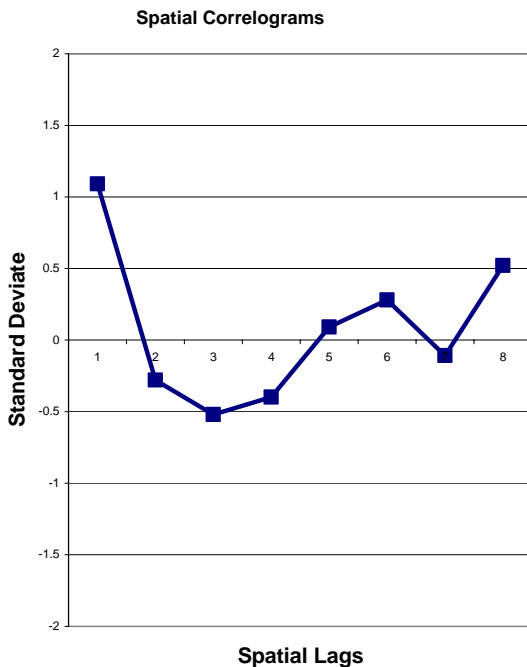
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Definition: In time-series it is common to look for at the autocorrelation structure for lags 1,2, 3,..., n time periods. The spatial analogue considers the distance or lags between pairs of geographical units (i.e., states, counties, census tracts, census blocks). For example, assuming the rook's case definition of adjacency (i.e., only shared boundaries; rows and columns) if county A shares a boundary with four other counties (B, C, D, and E) then these counties have a spatial lag of 1 (see diagram). Counties B, C, D, and E all have additional neighbors; F,G, H, I, J, K, L, and M. These eight counties are all two spatial lags from County A. Higher-order lags do not permit circular routes (i.e., A to B is a first spatial lag not a third spatial lag via C&M or D&G).

		F 2		
	M 2	B 1	G 2	
L 2	C 1	A	D 1	H 2
	K 2	E 1	I 2	
		J 2		

Moran's I is a measure of spatial autocorrelation. $I(k)$ is Moran's statistic, where k is the number of spatial lags between a set of paired geographical units (e.g., counties). A spatial correlogram is a plot of $I(k)$ against the number of spatial lags, k for $k = 1, 2, \dots, n$. As k increases, the value of the spatial autocorrelation coefficient I should decrease, since increasing lags diminishes correlation.

Application: The example below is of measles epidemics in South-West England (Cliff et al., 1975, p. 167-173).



The general pattern is one of positive spatial autocorrelation at lags 1, 6, and 8 and negative spatial autocorrelation at 2, 3, and 4. The positive spatial autocorrelation at lag 1 and the negative spatial autocorrelation 2, 3, and 4 suggest that the measles outbreaks are clustered spatially. If a region has an outbreak/no outbreak, contiguous regions are likely to behave similarly. In the example, the spatial lags 1 and 2 were dominated by urban-rural and rural-rural links, whereas lags 4-8 are dominated by urban-urban links. In addition, lags 6-8 were predominantly regions linked in an east-west direction. The correlogram analysis suggests similar levels of measles cases in non-contiguous urban areas, and contiguous rural-urban and rural-rural areas. A possible explanation of this pattern is initial outbreaks of measles in urban areas in an epidemic, hence the positive spatial autocorrelation at lags 6 and 8. This could be a central place effect. This is followed by a spread of the disease from the urban areas to the surrounding rural areas by a spatial diffusion process.

References/Sources:

Andrew D. Cliff, Peter Haggett, J. Keith Ord, Keith A. Bassett, and Richard B. Davies. 1975. Elements of Spatial Structure: A Quantitative Approach. Cambridge University Press: Cambridge, United Kingdom.